



# Bar Modelling



September 2020



## **Introduction**

Bar modelling is used as an effective part of the Concrete, Pictorial, Abstract (CPA) approach to the mastery of mathematics. Concrete materials are embedded alongside pictorial representations and abstract expressions to ensure procedural fluency and conceptual understanding are developed in tandem. It is not a method for problem solving but does reveal the mathematical structure beneath the problem and mathematical relationships between its component parts. Many teachers have found the approach very effective in gaining a deep understanding for children as it provides many tangible and memorable experiences to draw upon in their learning.



# Year 1

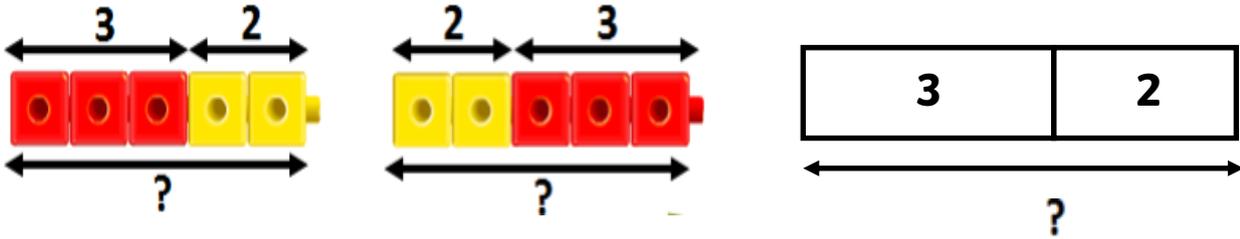


## Bar Modelling

### Addition

Example:

Three yellow sweets and two red sweets are on the table. How many sweets are on the table?



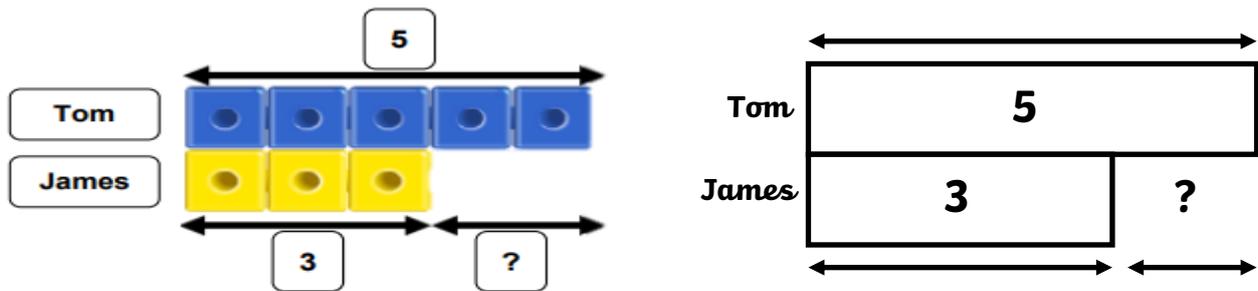
Three and two more is equal to five.  
Three add two equals five.  
Two and three more is equal to five.  
Two add three equals five.

$$3 + 2 = 5$$
$$5 = 2 + 3$$
$$2 + 3 = 5$$
$$5 = 3 + 2$$

### Subtraction

Example:

Tom has five sweets and James has three sweets. How many more sweets does Tom have than James?



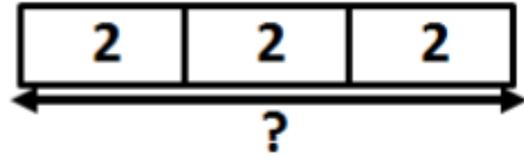
Five is two more than three.  
Three is two fewer than five.

$$5 = 3 + 2$$
$$5 - 3 = 2$$

## Multiplication

Example:

Amber wants to give her three friends two lollies each. How many lollies does she need?



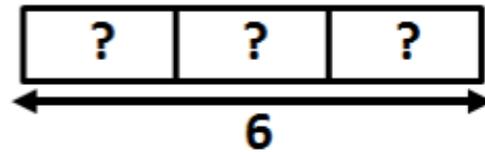
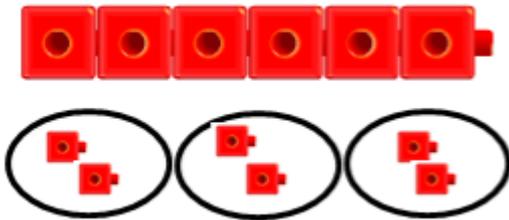
Two and two and two makes six.  
Three groups of two make six.

$$2 + 2 + 2 = 6$$
$$3 \times 2 = 6$$

## Division

Example:

Amber has six lollies. She wants to share them equally between her three friends. How many lollies does each of her friends get?



Six shared into three equal groups is two in each group.

$$6 \div 3 = 2$$



# Year 2

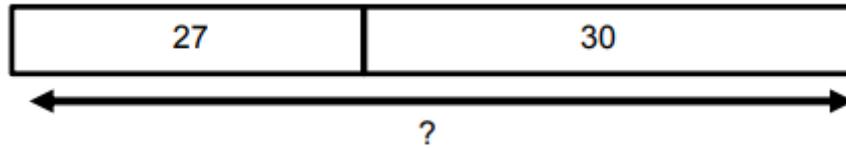


## Bar Modelling

### Addition

Example:

Jenny has 27 felt-tip pens. She buys 30 more. How many felt-tip pens does she now have?



30 more than 27 is 57.

57 is 27 more than 30.

27 more than 30 is 57.

30 fewer than 57 is 27.

$$27 + 30 = 57$$

$$30 + 27 = 57$$

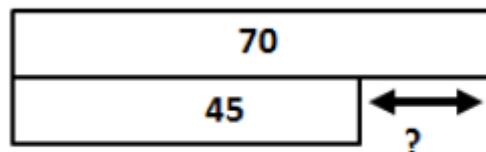
$$57 - 27 = 30$$

$$57 - 30 = 27$$

### Subtraction

Example:

Harry needs 70g of flour for his cake. There is 45g of flour left in the bag. How much more flour does he need?



25 more than 45 is 70.

70 is 45 more than 25.

25 less than 70 is 45.

45 fewer than 70 is 25.

$$70 = 45 + 25$$

$$45 + 25 = 70$$

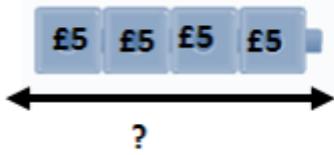
$$70 - 25 = 45$$

$$70 - 45 = 25$$

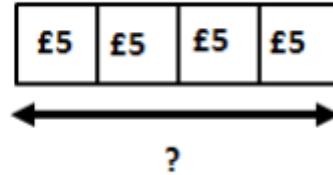
## Multiplication

Example:

James buys four tickets for the football match. Each ticket costs £5. How much does he spend?



Four groups of £5 is £20.  
£5 four times is £20.

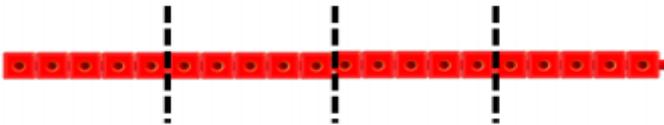


$£5 + £5 + £5 + £5 = £20$   
 $4 \times £5 = £20$

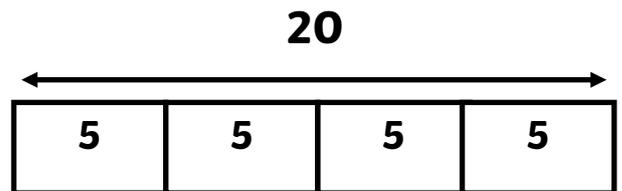
## Division

Example:

Claire wants to put 5 biscuits on each plate. She has 20 biscuits. How many plates does she need?



There are 4 groups of 5 in 20.



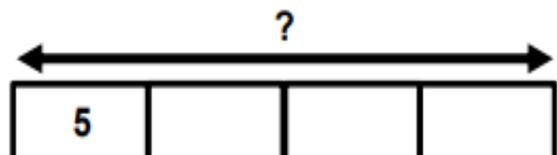
$20 \div 5 = 4$   
 $4 \times 5 = 20$

## Fractions

Example:

One quarter of a number is 5, what is the number?

Quarters



$4 \times 5 = 20$   
 $5 + 5 + 5 + 5 = 20$   
 $20 \div 4 = 5$



# Year 3

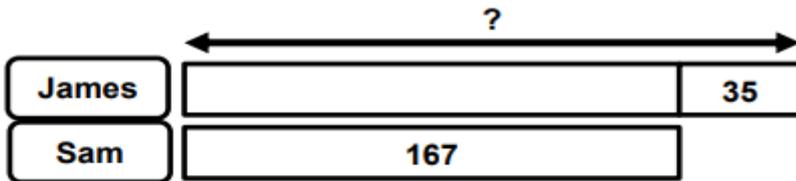


## Bar Modelling

### Addition and Subtraction

Example:

James scored 35 more points than Sam. Sam scored 167 points in his game. How many points did James score?



$$167 + 35 = 202$$

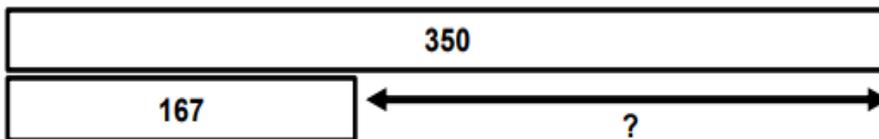
$$35 + 167 = 202$$

$$202 - 167 = 35$$

$$202 - 35 = 167$$

Example:

There are 350 pages in Debbie's book. On Tuesday, she reads 167 pages of her book. On Wednesday, she reads the rest of the book. How many pages did she read on Wednesday?



$$350 - 167 = 183$$

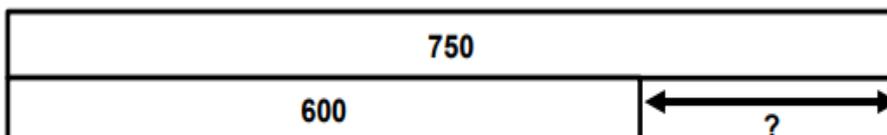
$$350 - 183 = 167$$

$$167 + 183 = 350$$

$$183 - 167 = 350$$

Example:

The bag of flour weighs  $\frac{3}{4}$  kg. Nicola uses 600g of flour. How much flour is left?



$$750\text{g} - 600\text{g} = 150\text{g}$$

$$750\text{g} - 150\text{g} = 600\text{g}$$

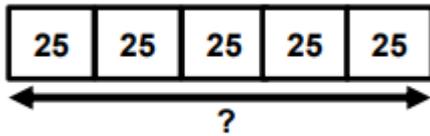
$$600\text{g} + 150\text{g} = 750\text{g}$$

$$150\text{g} + 600\text{g} = 750\text{g}$$

## Multiplication and Division

Example:

One length of the swimming pool is 25 metres. Rachel swims five lengths of the pool. How far does Rachel swim altogether?



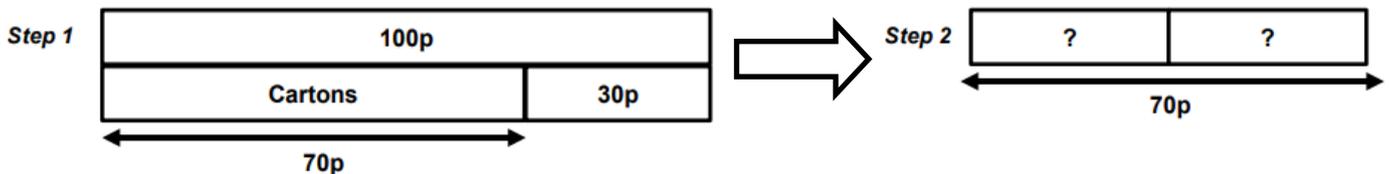
$$25 + 25 + 25 + 25 + 25 = 125$$

$$25 \times 5 = 125$$

$$5 \times 25 = 125$$

Example:

I had £1. I bought two cartons of drink and received 30p change. How much did each carton cost?



**Step 1**

$$100p - 30p = 70p$$

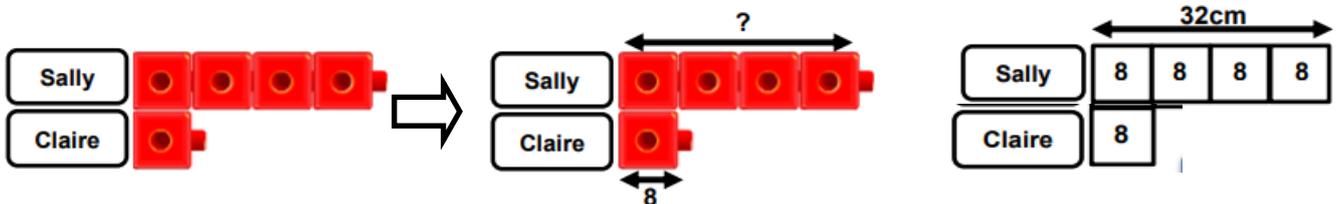
**Step 2**

$$70p \div 2 = 35p$$

$$35p + 35p = 70p$$

Example:

Claire builds a tower that is 8 cubes tall. Sally builds a tower that is 4 times as tall. How tall is Sally's tower? How much smaller is Claire's tower?



**How tall is Sally's tower?**

$$8 \times 4 = 32\text{cm}$$

$$4 \times 8 = 32\text{cm}$$

$$8 + 8 + 8 + 8 = 32\text{cm}$$

**How much smaller is Claire's tower than Sally's tower?**

$$32 - 8 = 24\text{cm}$$

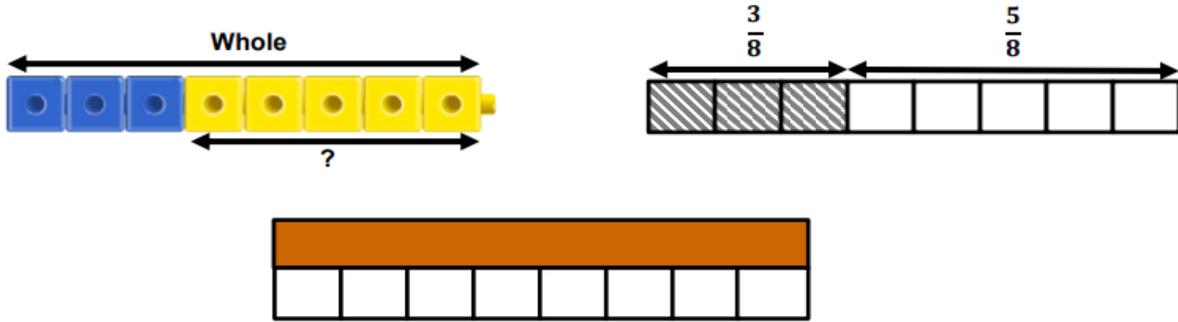
$$8 + 8 + 8 = 24\text{cm}$$

$$3 \times 8 = 24\text{cm}$$

**Fractions**

Example:

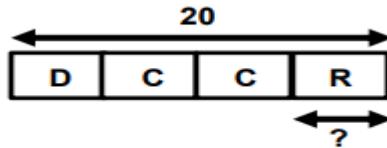
**3/8 of a class are boys. What fraction of the class are girls?**



3/8 of the class are boys, so 5/8 of the class will be girls.  
These fractions combined make a whole.

Example:

**Sally has 20 stickers on her page. One quarter of them are dog stickers. One half of them are cat stickers. The rest are rabbit stickers. How many rabbit stickers are on the page?**



The whole number 20 has been divided into four equal parts, or quarters.

$20 \div 4 = 5$

Dog stickers:  $1/4$  of 20 = 5

Cat stickers :  $1/2$  of 20 = 10

Rabbits stickers:  $1/4$  of 20 = 5

**$10 + 5 + 5 = 20$**

Example:

**What is the larger amount, one third of £60 or one quarter of £80?**



$60 \div 3 = 30$

$80 \div 4 = 20$

One third of £60 is the larger amount.



# Year 4

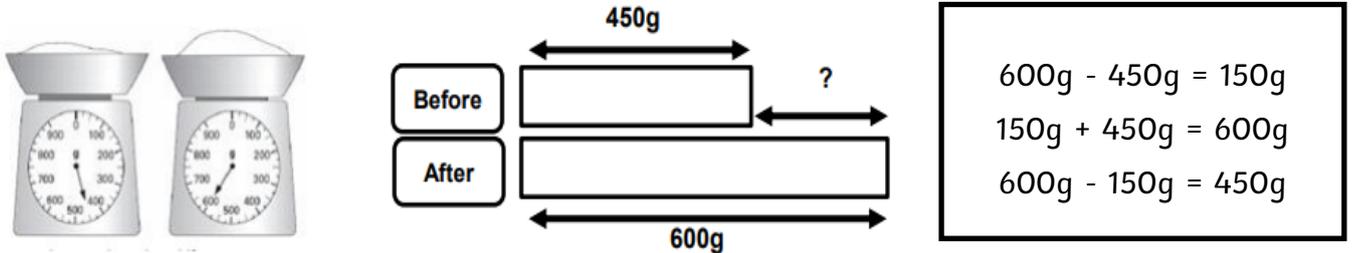


## Bar Modelling

### Addition and Subtraction

Example:

Emily is making a cake. She puts flour on the scales. How much sugar does she add?



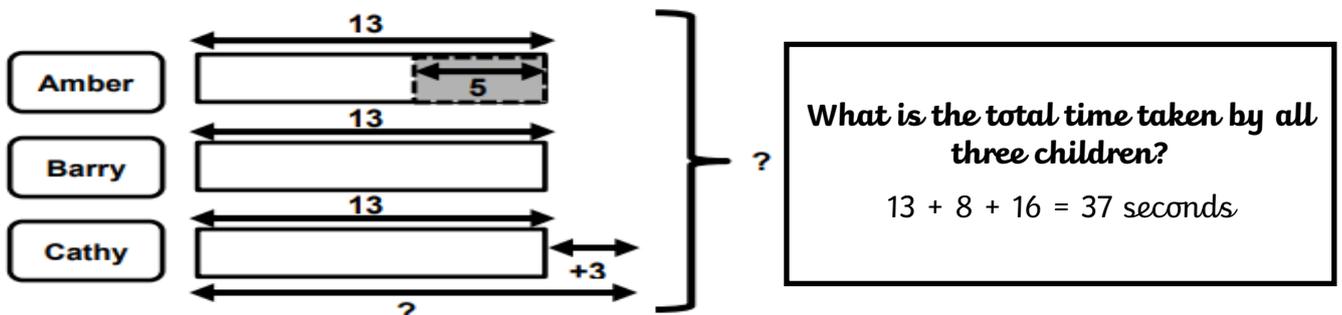
Example:

Claire has £1.37. She needs £1.38 more to buy a packet of pens. How much do the pens cost?



Example:

Amber, Barry and Cathy run a 50m race. Barry's time is 13 seconds. Amber finishes 5 seconds before Barry. Cathy finishes 3 seconds after Barry. What are each of their times? What is the total time taken by all three children?



**Barry's time:** 13 seconds

**Amber's time:** 13 seconds - 5 seconds = 8 seconds

**Cathy's time:** 13 seconds + 3 seconds = 16 seconds

**Multiplication and Division**

Example:

Louise collects buttons. She bought some new buttons to add to her collection. She now has 120 buttons, which is three times as many buttons as she started with. How many buttons did she start with?

The diagram shows a box labeled 'Buttons' containing three blue buttons. A double-headed arrow above the buttons is labeled 'Start value'. A longer double-headed arrow below the buttons is labeled 'Scaled value'. To the right, a row of three empty boxes is shown with a double-headed arrow above them labeled '?' and a double-headed arrow below them labeled '120'.

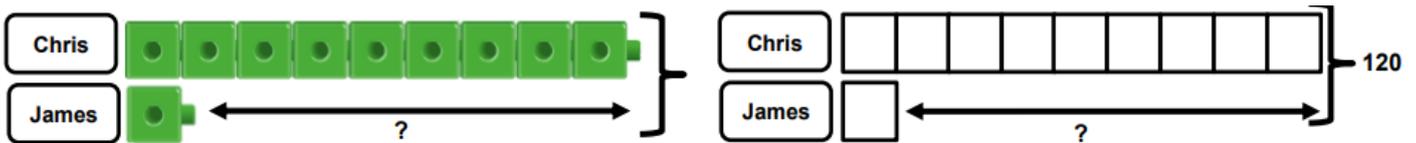
$$120 \div 3 = 40$$

$$40 + 40 + 40 = 120$$

$$40 \times 3 = 120$$

Example:

Chris has 9 times as many Lego figures as James. Together they have 120 Lego figures. How many more Lego figures does Chris have than James? How many would Chris have to give James so that they have an equal amount?



$$120 \div 10 = 12$$

Each block is worth 12 Lego figures

$$12 \times 9 = 108$$

**Chris has 108 more Lego figures than James.**

**How many Lego figures would Chris have to give James so that they have an equal amount?**

$$4 \times 12 = 48$$

$$12 + 12 + 12 + 12 = 48$$

Example:

An ice cream costs £1.20. Louise wants to buy 6 ice creams. How much will the ice creams cost?

The diagram shows a single green block with a double-headed arrow above it labeled 'One item'. Below it, a row of six green blocks is shown with a double-headed arrow below them labeled 'Six items'. To the right, a row of six empty boxes is shown with a double-headed arrow above the first box labeled '£1.20' and a double-headed arrow below them labeled '?'.

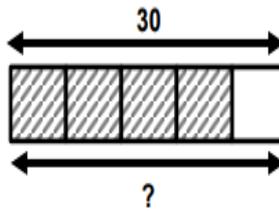
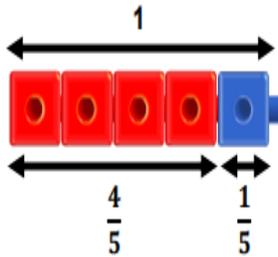
$$£1.20 \times 6 = £7.20$$

$$120p \times 6 = 720p$$

## Fractions

Example:

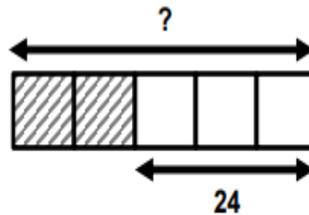
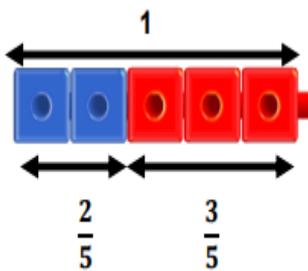
Sally buys four fifths of the shop's apples. If the shop had 30 apples, how many apples did she buy?



$30 \div 5 = 6$  ( $1/5$  of 30 is 6)  
 $6 \times 4 = 24$  ( $4/5$  of 30 is 24)  
Sally buys 24 apples.

Example:

James had some football cards. He gave two fifths away. He now has 24 cards. How many did he have to start with?



**Step 1:**  $24 \div 3 = 8$   
Each block is worth 8 cards.  
**Step 2:**  $8 \times 5 = 40$   
James started with 40 cards.



# Year 5



## Bar Modelling

### Multiplication and Division

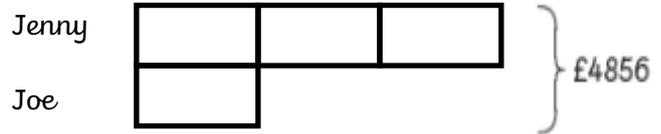
Example:

Jenny and Joe share the sum of £3597 and £1259 so that Jenny gets 3 times as much as Joe. How much more than Joe will Jenny get?

**Step 1:**



**Step 2:**



**Step 1:**  $£3597 + £1259 = £4856$

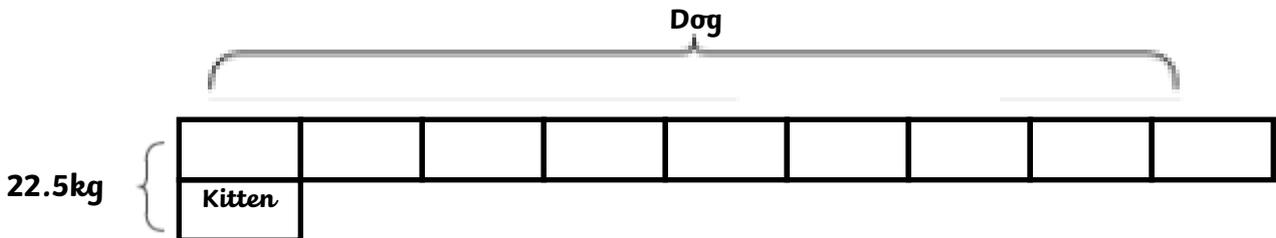
**Step 2:**  $£4856 \div 4 = £1214$

**Step 3:** Jenny:  $£1214 \times 3 = £3642$  / Joe =  $£1214$

**Step 4:** Jenny gets  $£3642 - £1214 = £2428$  more than Joe

Example:

A vet weighs a large dog and a kitten. The large dog weighs 9 times more than the kitten. Altogether they weigh 22.5kg. What does the dog weigh?



**Step 1:**  $22.5\text{kg} \div 10 = 2.25\text{kg}$

The kitten weighs 2.25kg

**Step 2:**  $2.25\text{kg} \times 9 = 20.25\text{kg}$

**Step 3:** Check the total:  $20.25\text{kg} + 2.25\text{kg} = 22.5\text{kg}$

The dog weighs 20.25kg

**Solving problems using addition, subtraction, multiplication and division and a combination of these, including understanding the equals sign**

Example:

Every day, for 4 days, Sally scored 12.5 on her test. On her fifth day, she scored 14. What was her total score for the week?

o Step 1

o Step 2

$$12.5 \times 4 = 50$$

$$50 + 14 = 64$$

Sally's total score was 64.

Example:

I cut 80cm from a length of ribbon and shared the remainder between 3 friends. Each friend now has 1.3m of ribbon. How much did I start with?

Step 1

Step 2

**Step 1:**  $130\text{cm} \times 3 = 390\text{cm}$  Or  $130\text{cm} + 130\text{cm} + 130\text{cm} = 390\text{cm}$

**Step 2:**  $390\text{cm} + 80\text{cm} = 470\text{cm}$  or  $4.7\text{m}$

Example:

At the school disco, there are 5 girls to every 3 boys. If there are 136 children at the disco, how many more girls than boys are there?

**Step 1:**  $136 \div 8 = 17$

**Step 2:**  $17 \times 2 = 34$

There are 34 more girls than boys.

**Fractions**

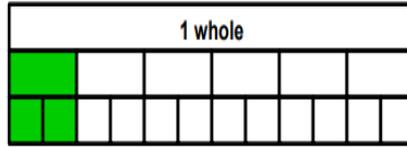
Example:

**Which fraction is greater  $3/4$  or  $1/6$  ?**

Three quarters converted into twelfths



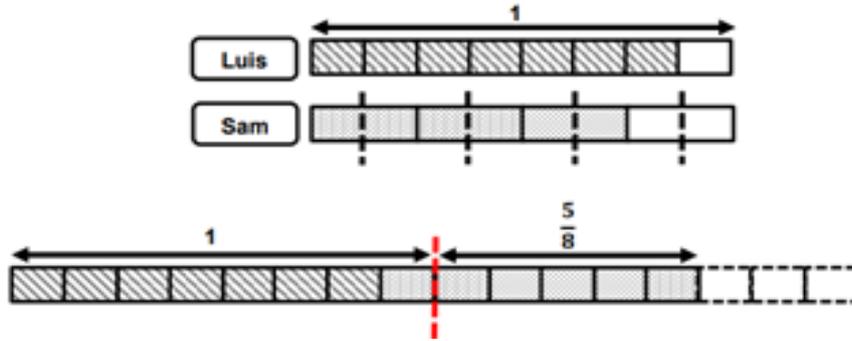
One sixth converted into twelfths



$3/4$  is the same as  $9/12$   
 $1/6$  is the same as  $2/12$   
 $3/4$  is greater than  $1/6$

Example:

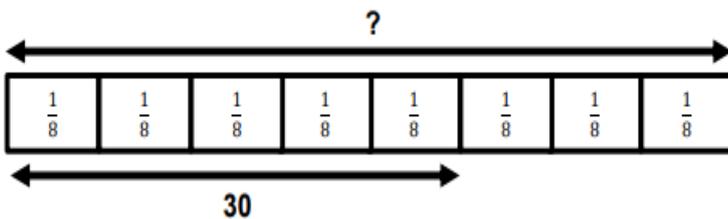
**Sam and Luis have a pizza each. Sam eats  $3/4$  of his pizza. Luis eats  $7/8$  of his pizza. How much pizza have they eaten altogether?**



$7/8 + 3/4 = ?$   
 $3/4$  is equivalent to  $6/8$   
 $7/8 + 6/8 = 13/8 = 1 \frac{5}{8}$   
 They have eaten  $1 \frac{5}{8}$  of the pizza.

Example:

**30 is  $5/8$  of a number. What is the number?**



$30 \div 5 = 6$  ( $1/8$  of the number is 6).  
 $8 \times 6 = 48$   
 The number is 48



# Year 6

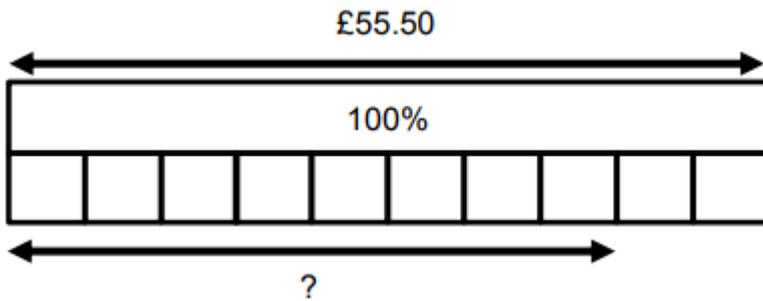


## Bar Modelling

### Percentages

Example:

The normal price for a tracksuit is £55.50. How much will it cost in a '20% off' sale?



**Step 1:** What is 10%?

$$£55.50 \div 10 = £5.55$$

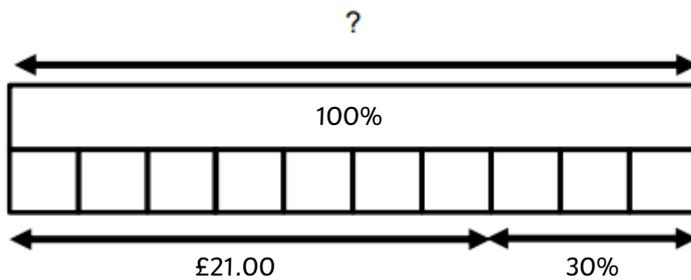
**Step 2:**  $£5.55 \times 2 = £11.10$

**Step 3:**  $£55.50 - £11.10 = £44.40$

It will cost £44.40 in the sale.

Example:

In a '30% off' sale, Carl pays £21 for a T-shirt. How much was the T-shirt before the sale?



**Step 1:**  $£21.00 \div 7 = £3.00$

**Step 2:**  $£3.00 \times 3 = £9.00$

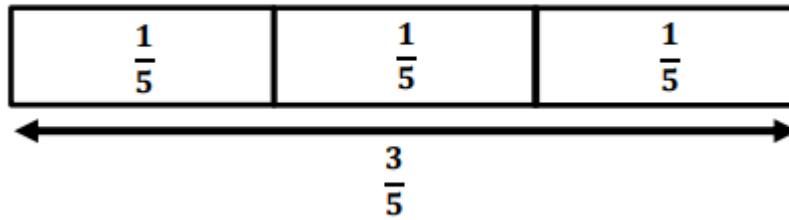
**Step 3:**  $£21.00 + £9.00 = £30.00$

The t-shirt cost £30 before the sale.

## Divide proper fractions by whole numbers

Example:

$$3/5 \div 3 =$$



**What can you see?**

*3/5 shared between 3 people would be 1/5 each.*

*I can see that 1/5 is 1/3 of 3/5 .*

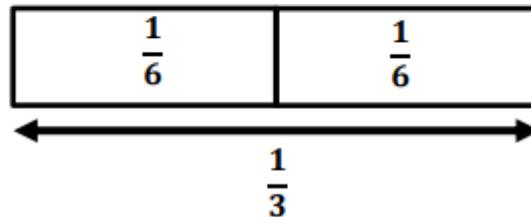
*I can see that 3 x 1/5 is equal to 3/5 .*

*I can see how many fifths are in three fifths.*

$$3/5 \div 3 = 1/5$$

Example:

$$1/3 \div 2 =$$



**What can you see?**

*I can see that one third is the same as two sixths.*

*I can see that half of one third is one sixth.*

*I can see double one sixth is equal to one third.*

*I can see two sixths are equivalent to one third.*

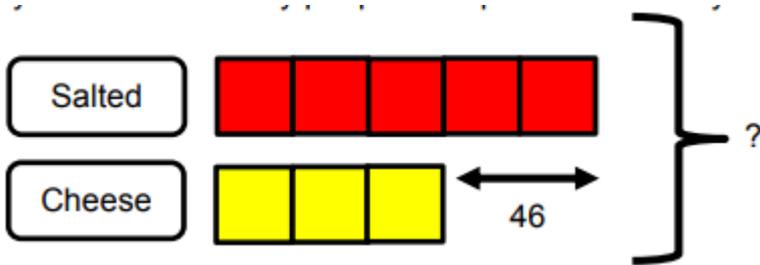
*I can see that there are two sixths in a third.*

$$1/3 \div 2 = 1/6$$

Unequal sharing and grouping using knowledge of fractions and multiples

Example:

In a survey, the ratio of the number of people who preferred 'ready-salted' to 'cheese and onion' crisps was 5:3. Forty-six more people preferred ready-salted. How many people took part in the survey?



**Step 1:**  $46 \div 2 = 23$

**Step 2:** Ready Salted:  $5 \times 23 = 115$

Cheese and Onion:  $3 \times 23 = 69$

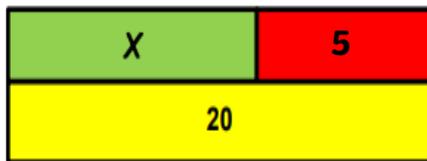
**Step 3:**  $115 + 69 = 184$

184 people took part in the survey.

Express missing number problems algebraically

Example:

$X + 5 = 20$



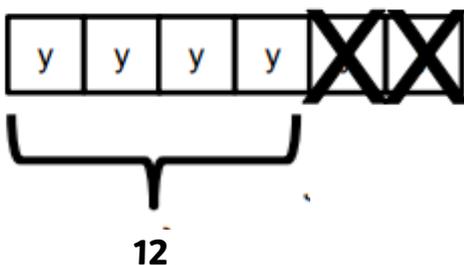
$20 - 5 = 15$

$X = 15$

Example:

$6y - 2y = 12$

$y = ?$

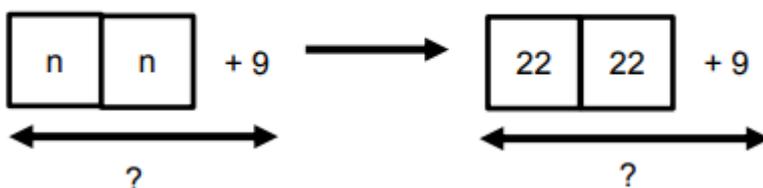


$12 \div 4 = 3$

$y = 3$

Example:

What is  $2n + 9$  when  $n = 22$ ?



**Step 1:**  $22 + 22 = 44$

**Step 2:**  $44 + 9 = 53$

$2n + 9 = 53$